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The Design of Partly Grooved, Externally Pressurized Journal Bearings

The equation for the pressure buildup in these bearings is subjected to a first-order perturbation with respect to parallel as well as skewed displacements of the center lines of journal and bearing. Fluid film forces and their points of application can thus be found. Several configurations of these bearings are discussed. One configuration acts as a simple support, and two others act as a clamped support to the rotatable journal. Promising applications in machines are also discussed.

Introduction

ADAMS [1][1] and Mannan, et al. [2] have described two types of externally pressurized journal bearings, in which external restrictions around the circumference are superfluous.

These two bearings types are depicted in Figs. 1 and 2, respectively. The outflowing fluid is subjected to narrowing passages. This feature provides a stable equilibrium of external forces and resultant pressures in the fluid film. In an earlier paper of the author [3] a further development of the bearing in Fig. 1 has been introduced. It is depicted in Fig. 3.[2] The outflowing fluid is again subjected to narrowing passages. Moreover the grooves in the new type only offer a liberal passage to axially flowing fluid; they keep the passage to short-circuiting flows around the circumference as small as possible. Thus short-circuiting flows in circumferential direction are suppressed. Accordingly, the load capacity, stiffness, and power consumption of the grooved bearing type must be more favorable than of the ungrooved type of Figs. 1 and 2.

[1] Numbers in brackets designate References at end of paper.

[2] Patents pending.

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In the earlier paper [3], fluid film forces such as result from a first order perturbation with respect to a parallel displacement of the center lines of journal and bearing, as well as power consumption at a concentric position have been provided. Much attention has been devoted to comparing these properties for the grooved bearing type and the conventional type incorporating external restrictions.

The present paper forms an extension to the previous one in that it includes:

1 Fluid film forces and their points of application such as result from a first order perturbation with respect to parallel displacements of the center lines of journal and bearing.

2 Fluid film forces and their points of application such as result from a first order perturbation with respect to the misalignment angle between the center lines of journal and bearing.

Several unique properties of the grooved bearing type evolve. Grooved bearings can be applied easier and at less cost than conventional types with external restrictions. Moreover, the number of applications possible is greater for the grooved type.

Pressure Buildup

In [3], [4], and [5] the pressure buildup in grooved bearings has been studied extensively. For incompressible lubricants, the

Nomenclature

$(\phi_l)_m$ = mean mass flow across ridge and groove in axial direction

$\phi_{\varphi r}$ = mass flow per unit width in circumferential direction

ρ = density

η = viscosity

p = pressure

l = axial coordinate

φr = circumferential coordinate

r = radius

h = film thickness

$(h^3)_m \left(\frac{1}{h^3}\right)_m^{-1}$ = equivalent passages to axial and circumferential flow, respectively

h_0 = radial clearance as measured on a ridge or an ungrooved zone

γ = ratio of groove width to groove + ridge width

δ = ratio of film thickness in a groove to that on a ridge if the journal is concentric with respect to the bearing

ϵ = ratio of eccentricity to radial clearance

β = misalignment angle

λ, φ = dimensionless coordinates in axial and circumferential direction, respectively $\left(\lambda = \frac{l}{r}\right)$

Δp = excess-pressure

p_m = mean pressure per unit projected bearing area

Q = power consumption for configurations A and C in Fig. 7

ϕ_t = volume flow for configurations A and C in Fig. 7

λ_0 = length of grooved + ungrooved zone divided by radius

λ_1 = length of ungrooved zone divided by radius

λ_2, λ_3 = dimensionless distances from free edge of ungrooved zone to points of application of resultants of the pressures due to parallel and skewed displacements, respectively

λ_4 = dimensionless width of circumferential groove

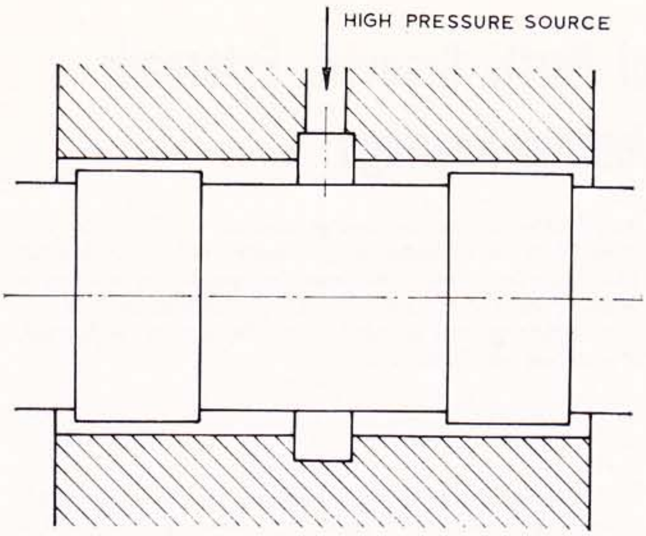

Fig. 1 Stepped journal bearing

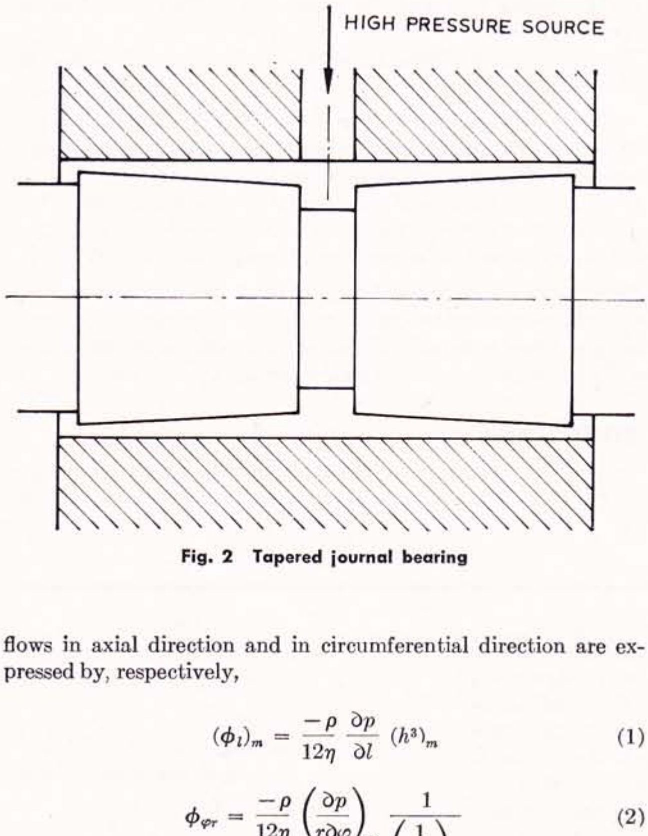

Fig. 2 Tapered journal bearing

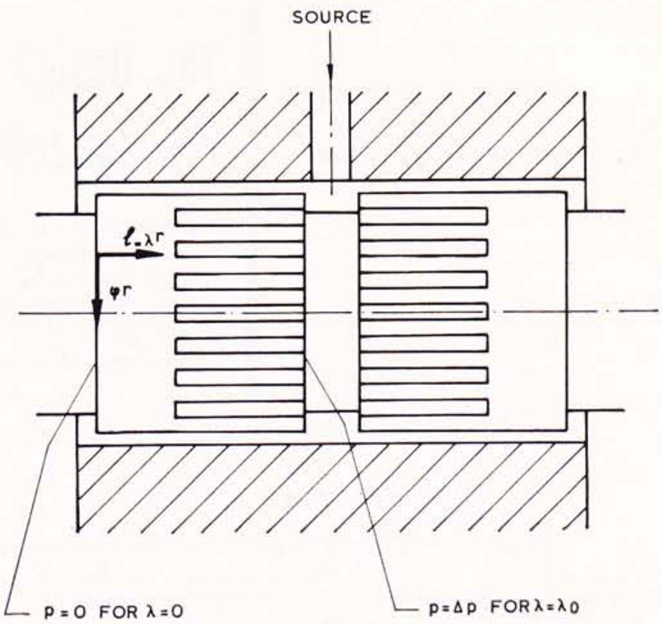

Fig. 3 Partly grooved journal bearing

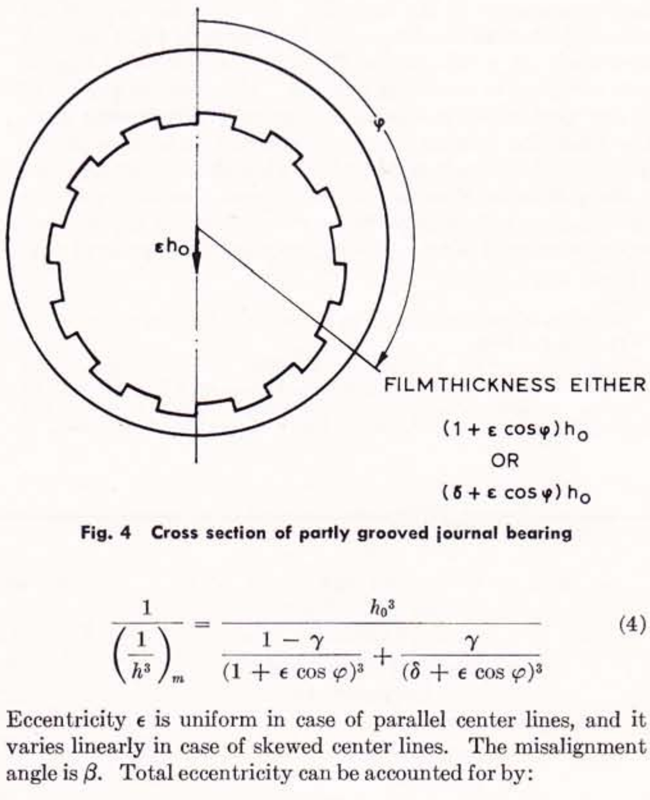

Fig. 4 Cross section of partly grooved journal bearing

flows in axial direction and in circumferential direction are expressed by, respectively,

$$(\phi_l)_m = \frac{-\rho}{12\eta}\frac{\partial p}{\partial l}(h^3)_m \tag{1}$$

$$\phi_{\varphi r} = \frac{-\rho}{12\eta}\left(\frac{\partial p}{r\partial\varphi}\right)_m \frac{1}{\left(\frac{1}{h^3}\right)_m} \tag{2}$$

Quantities within brackets indicate that their wave-like component has been averaged out over a ridge and a groove. This procedure, which may be applied if the number of grooves is sufficiently great, is based on connecting passages to axial flow in parallel, and on connecting passages to circumferential flow in series.[3]

The film thicknesses in a cross section of journal and bearing are depicted in Fig. 4. The passages connected in parallel and in series can now be specified:

$$(h^3)_m = \left\{(1-\gamma)(1+\epsilon\cos\varphi)^3 + \gamma(\delta+\epsilon\cos\varphi)^3\right\}h_0^3 \tag{3}$$

[3] Passage to plane Poiseuille flow is here defined as film thickness to the third power.

$$\frac{1}{\left(\frac{1}{h^3}\right)_m} = \frac{h_0^3}{\frac{1-\gamma}{(1+\epsilon\cos\varphi)^3} + \frac{\gamma}{(\delta+\epsilon\cos\varphi)^3}} \tag{4}$$

Eccentricity ϵ is uniform in case of parallel center lines, and it varies linearly in case of skewed center lines. The misalignment angle is β. Total eccentricity can be accounted for by:

$$\epsilon = \epsilon_{\text{parallel}} + \beta\lambda, \quad \text{where} \quad \lambda = \frac{l}{r} \tag{5}$$

Film pressures can now be obtained by inserting equations (1) and (2) into the following equation, which accounts for the continuity of flow

$$\frac{\partial(\phi_l)_m}{\partial l} + \frac{\partial\phi_{\varphi r}}{r\partial\varphi} = 0 \tag{6}$$

and by solving the resulting differential equation. Boundary conditions are specified in Fig. 3.

Ausman [6] and [7] has shown that first order perturbation methods with respect to uniform eccentricity and misalignment angle, respectively, are useful in bearing analysis. The present author has applied this method in [3] and [4]. No attention will be devoted to it in the present paper. Let it suffice to indicate that dependent quantities, for instance load capacity, evolve in the following form:

Dependent quantity = independent quantity, determined by groove and bearing parameters, times either eccentricity ϵ or misalignment angle β plus higher order terms of ϵ or β

and that the higher order terms are omitted. Mannan's results [2], for the bearing of Fig. 2, show that the first order term is dominant, and that at maximum eccentricity ($\epsilon = 1$), higher order terms contribute less than 20 percent to the load capacity. Thus it is inferred that the higher order terms are negligibly small for the grooved bearing of Fig. 3 also.

Criteria for Optimizing Groove Parameters

In [3], three aims for optimizing have been introduced.

1 Maximum load capacity and stiffness at a given excess pressure:

$$\left(\frac{p_m}{\epsilon \cdot \Delta p}\right)_{\max}$$

2 Maximum load capacity and stiffness at a given power consumption:

$$\left(\frac{p_m{}^2 h_0{}^3}{\epsilon^2 Q \eta}\right)_{\max}$$

3 Maximum load capacity and stiffness at a given throughflow:

$$\left(\frac{p_m h_0{}^3}{\epsilon \phi_t \eta}\right)_{\max}$$

The dimensionless groups or numbers within brackets depend on dimensionless groove parameters γ and δ and on the axial length of a plain and a grooved region in dimensionless form, λ_1, and $\lambda_0 - \lambda_1$, respectively. In the optimization process, the parameter γ has to be provided with a lower limit in order to avoid unpractically narrow (and deep) grooves evolving ($\gamma = 0.1$ 0.2, 0.3, and 0.5).

Moreover the extent of the ungrooved region has to be provided with a lower limit acceptable to practice ($\lambda_1 = 0.1$) while maximizing number:

$$\frac{p_m}{\epsilon \cdot \Delta p}$$

The groove and bearing parameters are given in Table 1; see columns 1, 2, 3, and 4. Underlined quantities in columns 5, 9, and 10 indicate which dimensionless number has been optimized. The other numbers have been computed using the groove parameters which have been found in the optimization. The meaning of columns 6, 7, and 8 will be given in the next section.

New Quantities

The magnitude of the three numbers mentioned in the foregoing and the related groove and bearing parameters are not yet sufficient to design these grooved bearings successfully in applications where the misalignment might cause the center lines of journal and bearing to become skewed. Such applications are exemplified by short shafts with overhung rotors. Thus the resultant of the pressures and its point of application should be known for uniform as well as nonuniform eccentricities. Columns 5, 6, 7, and 8 in Table 1 are useful for determining these quantities.

Column 5 gives the already familiar number $\frac{p_m}{\epsilon \cdot \Delta p}$ obtaining with uniform eccentricity ϵ. Column 6 gives a number having the character of a dimensionless bending moment $\frac{p_m \lambda_2}{\epsilon \cdot \Delta p}$, it also occurs with uniform eccentricity ϵ. The dimensionless length λ_2 extends from the free edge of the ungrooved region to the point of application, see Fig. 5. Column 7 gives the number $\frac{p_n}{\beta \cdot \Delta p}$ occurring with nonparallel eccentricities $\epsilon = \beta\lambda$, see Fig. 6. Column 8 gives a number having the character of a bending moment $\frac{p_n \lambda_3}{\beta \cdot \Delta p}$, it also obtains with nonuniform eccentricity. The dimensionless length λ_3 extends from the free edge of the ungrooved zone to the point of application, see Fig. 6.

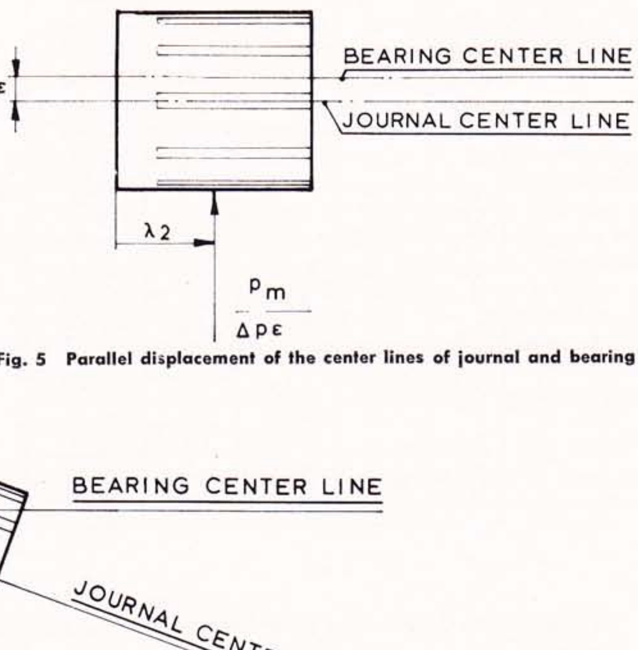

Fig. 5 Parallel displacement of the center lines of journal and bearing

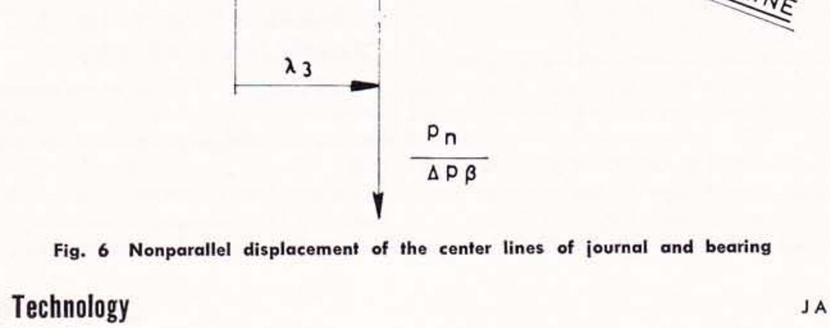

Fig. 6 Nonparallel displacement of the center lines of journal and bearing

Table 1 Design data for partly grooved externally pressurized bearings

γ	δ	λ_1	λ_o	$\frac{p_m}{\epsilon \cdot \Delta p}$	$\frac{p_m \lambda_2}{\epsilon \cdot \Delta p}$	$\frac{p_n}{\beta \cdot \Delta p}$	$\frac{p_n \lambda_3}{\beta \cdot \Delta p}$	$\frac{p_m^2 h_o^3}{\epsilon^2 Q \eta}$	$\frac{p_m h_o^3}{\epsilon \phi_t \eta}$
0.1	4.94	0.100	1.0	-0.415	-0.152	0.0289	0.0124	0.0278	0.0668
0.1	3.53	0.216	1.0	-0.325	-0.131	0.0415	0.0184	0.0367	0.113
0.1	2.89	0.355	1.0	-0.243	-0.110	0.0500	0.0219	0.0310	0.128
0.2	3.98	0.100	1.0	-0.390	-0.143	0.0340	0.0145	0.0243	0.0621
0.2	2.90	0.207	1.0	-0.306	-0.123	0.0463	0.0205	0.0309	0.101
0.2	2.37	0.351	1.0	-0.225	-0.101	0.0539	0.0237	0.0259	0.116
0.3	3.52	0.100	1.0	-0.374	-0.137	0.0370	0.0158	0.0221	0.0592
0.3	2.60	0.201	1.0	-0.294	-0.117	0.0490	0.0216	0.0276	0.0936
0.3	2.12	0.350	1.0	-0.212	-0.0953	0.0560	0.0247	0.0228	0.108
0.5	3.02	0.100	1.0	-0.350	-0.128	0.0411	0.0174	0.0190	0.0544
0.5	2.27	0.193	1.0	-0.275	-0.109	0.0527	0.0232	0.0232	0.0840
0.5	1.84	0.350	1.0	-0.194	-0.0869	0.0593	0.0263	0.0190	0.0984
0.1	6.23	0.100	2.0	-0.448	-0.312	0.0467	0.0391	0.0336	0.0754
0.1	3.76	0.384	2.0	-0.311	-0.244	0.0695	0.0612	0.0593	0.191
0.1	3.02	0.692	2.0	-0.220	-0.197	0.0807	0.0694	0.0487	0.221
0.2	5.00	0.100	2.0	-0.425	-0.296	0.0560	0.0466	0.0300	0.0707
0.2	3.11	0.363	2.0	-0.294	-0.229	0.0773	0.0678	0.0498	0.169
0.2	2.48	0.687	2.0	-0.202	-0.180	0.0865	0.0747	0.0402	0.199
0.3	4.41	0.100	2.0	-0.409	-0.285	0.0616	0.0511	0.0275	0.0668
0.3	2.79	0.350	2.0	-0.281	-0.217	0.0819	0.0717	0.0436	0.156
0.3	2.21	0.688	2.0	-0.189	-0.168	0.0901	0.0779	0.0349	0.184
0.5	3.79	0.100	2.0	-0.383	-0.266	0.0680	0.0562	0.0237	0.0621
0.5	2.46	0.328	2.0	-0.261	-0.199	0.0860	0.0749	0.0350	0.135
0.5	1.92	0.701	2.0	-0.167	-0.149	0.0927	0.0799	0.0273	0.163
0.1	7.14	0.100	3.0	-0.455	-0.467	0.0606	0.0751	0.0352	0.0773
0.1	4.11	0.481	3.0	-0.297	-0.336	0.0839	0.110	0.0674	0.227
0.1	3.20	1.000	3.0	-0.194	-0.256	0.0918	0.115	0.0528	0.273
0.2	5.74	0.100	3.0	-0.434	-0.445	0.0726	0.0895	0.0315	0.0726
0.2	3.40	0.449	3.0	-0.261	-0.315	0.0947	0.124	0.0561	0.200
0.2	2.62	0.998	3.0	-0.176	-0.232	0.0990	0.125	0.0431	0.245
0.3	5.06	0.100	3.0	-0.418	-0.428	0.0800	0.0985	0.0289	0.0688
0.3	3.07	0.423	3.0	-0.270	-0.300	0.100	0.131	0.0486	0.180
0.3	2.34	1.000	3.0	-0.163	-0.215	0.102	0.129	0.0366	0.224
0.5	4.36	0.100	3.0	-0.390	-0.398	0.0874	0.107	0.0245	0.0630
0.5	2.76	0.377	3.0	-0.252	-0.274	0.103	0.133	0.0372	0.148
0.5	2.02	1.040	3.0	-0.139	-0.184	0.104	0.129	0.0269	0.194
0.1	7.90	0.100	4.0	-0.455	-0.616	0.0715	0.117	0.0351	0.0773
0.1	4.52	0.515	4.0	-0.286	-0.415	0.0920	0.160	0.0671	0.235
0.1	3.39	1.280	4.0	-0.168	-0.290	0.0924	0.150	0.0496	0.295
0.2	6.36	0.100	4.0	-0.435	-0.587	0.0856	0.140	0.0315	0.0726
0.2	3.76	0.471	4.0	-0.274	-0.392	0.105	0.181	0.0557	0.203
0.2	2.78	1.280	4.0	-0.152	-0.263	0.0998	0.162	0.0399	0.263
0.3	5.61	0.100	4.0	-0.419	-0.565	0.0942	0.154	0.0289	0.0688
0.3	3.42	0.434	4.0	-0.265	-0.375	0.111	0.191	0.0477	0.181
0.3	2.48	1.290	4.0	-0.139	-0.240	0.103	0.167	0.0333	0.240
0.5	4.84	0.100	4.0	-0.389	-0.522	0.102	0.166	0.0243	0.0621
0.5	3.12	0.363	4.0	-0.250	-0.346	0.114	0.195	0.0354	0.142
0.5	2.14	1.370	4.0	-0.114	-0.199	0.103	0.162	0.0231	0.202

Table 1 shows the following general properties of the numbers in question.

1 Numbers characteristic of skewing as in Fig. 6 are an order of magnitude smaller than those due to uniform eccentricities.

2 Parallel displacements of the journal center line are directed oppositely to the resultant of the pressure, see Fig. 5; nonparallel displacements such as in Fig. 6 and the resultant of the pressures are directed the same way.

3 The point of application of the resultant of the pressure due to parallel displacements is always closer to the free edge than that due to nonparallel displacements as in Fig. 6 ($\lambda_2 \ll \lambda_3$).

4 The points of application of the two resultants of the pressures mentioned under item 3 are very close when compared with bearing length ($\lambda_3 - \lambda_2 \ll \lambda_0$).

Configurations of Partly Grooved, Externally Pressurized Bearings

Configurations of these bearings are enumerated systematically as regards the locations of sources and sinks with respect to the bearing surfaces, see Fig. 7. Configuration A has a source on either side and one central sink. The distance between the two free edges of the ungrooved zones can be adapted to requirements regarding the realigning moment of the bearing with respect to the rotatable journal. The realigning moment is positive for a

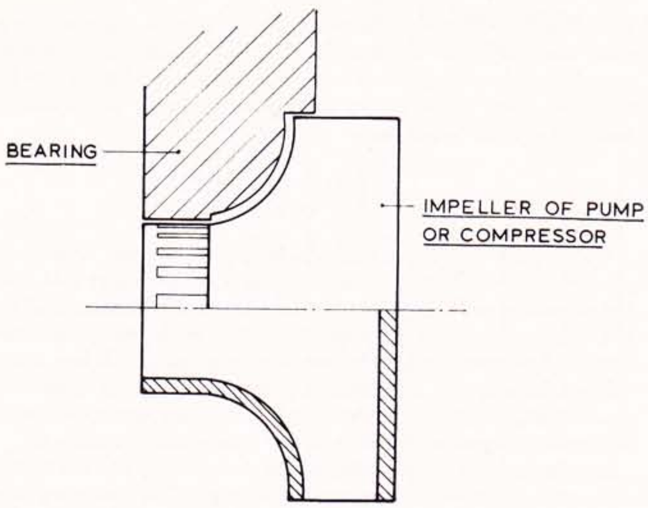

Fig. 10 Bearing for centrifugal compressor or pump

and where a small leakage across the piston is allowed. Configuration C is applied here in order to get a substantial realigning moment. It is evident that this design prevents that pistons or plungers jam in the cylinders. Thus, the present application is of importance to hydromotors, actuators, and many other devices in hydraulic systems. It is expected that the present design is more simple and less costly than the one using conventional, externally pressurized bearings such as described by Raimondi and Boyd [8].

3 Bearing for Centrifugal Compressors and Pumps. In many centrifugal pumps and compressors the fluid to be transported must also serve as a lubricant. More often than not the fluid has a low viscosity. In such cases, it is suggested to locate the journal bearing of configuration B, as in Fig. 10. Four advantages are claimed:

(*a*) The bearing acts as a seal. The leakage will be much smaller than in the conventional labyrinth-seal, thanks to much smaller clearances extending over much greater areas.

(*b*) The load capacity of the bearing is directly due to the difference in the pressures over the impeller.

(*c*) The bearing can be given a great diameter, while there is no need installing a great-diameter journal.

(*d*) In multistage pumps and compressors, each impeller can be provided with only one bearing. Thus, a stiff design, a thin shaft, and a spacious entrance per impeller are achieved simultaneously.

Conclusions

The data worked out in the present paper will prove helpful in the design of various configurations of partly grooved, externally pressurized bearings. These data are restricted to the effect of external pressurization, when using incompressible lubricants.

Acknowledgments

The work reported in this paper is part of a research program on grooved bearings and grooved seals carried out at the Institute TNO for Mechanical Constructions, Delft, Holland. The author wishes to thank Professor H. Blok for his advice and interest.

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DISCUSSION

C. R. Adams[4]

The author states in his opening sentence that external restrictions are superfluous in the Adams and Mannan bearing. Apparently, Mr. Hirs does not understand the operation of the stepped journal bearing. Analysis[5] and tests of the stepped bearing have shown the important functions of the step in controlling load capacity, stiffness, and flow rate.

The author states in his paper that by elimination of circumferential short-circuiting flows the grooved bearing automatically improves load capacity, stiffness, and power consumption. Grooving may reduce circumferential flows, but in so doing, it also may reduce load capacity and stiffness while making the bearing highly unstable.

Tests conducted on grooved air bearings to determine flow patterns showed circumferential flow when the ratio of film thickness in the groove to that on a ridge was large (greater than 2). Flow patterns on *properly designed* stepped air bearings showed very little circumferential flow. Groove depths determined by Hirs' analysis appear to be excessively large for a stable gas bearing design. It is interesting to note that in H. Arneson's patent,[6] the use of shallow precision grooves is shown in order that the fluid flowing along the grooves has a reduction in pressure due to viscous drag.

The author states that the grooved bearing is less costly than the conventional type. Costs of precision-machining concentric diameters for a stepped journal bearing are considerably less than costs of producing precision axial grooves.

It will be interesting to see a report on actual test results of gas bearings designed in accordance with the analysis.

H. Arneson[7]

Mr. Hirs has made a useful contribution to the art of grooved hydrostatic bearings by presenting a table which gives load capacity and stiffness over a wide range of applications.

A word of caution is in order regarding the stiffness number, $\frac{p_m}{\epsilon \cdot \Delta p}$. It gives a realistic value for the stiffness when the shaft is near center; however, as the eccentricity approaches 1 its value decreases. For example, Table 1 shows $\frac{p_m}{\epsilon \cdot \Delta p} = 0.455$ when $\gamma = 0.1$, $\delta = 7.90$, and $\lambda_0 = 4$; but as the eccentricity approaches 1.0, the actual load capacity in this case will be on the order of 0.31. In the design of gas bearings particularly, the problem is

[4] The Boeing Co., Aerospace Group, Space Division, Seattle, Wash.

[5] Adams, C. R., Dworski, J., and Shoemaker, E. M., "Externally Pressurized Step Journal Bearings," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 83, No. 4, Dec. 1961, pp. 595–602.

[6] U. S. Patent No. 3,305,282, "Hydrostatic Bearing Structure."

[7] Partner, Professional Instruments Co., Minneapolis, Minn.

often how to get enough load capacity within the limits of the available space and pressure. Consequently, the designer must work closer to the actual bearing capacity than is usual with conventional bearings. For this reason it would be useful to extend the table to give actual ultimate load capacities.

No criterion is given by which to determine when the number of grooves is sufficiently great for the application of the flow equations. A detailed study[8] of the interactions between the groove streams and their adjacent rib streams in a region of flow transition near the ends of the grooves has shown that stream convergence begins to be incomplete when the rib width exceeds four times the length of the ungrooved zone. Further, in a precision analysis, this flow transition region must be allowed for in the determination of the ratio of the length of the grooves to the length of the ungrooved portion $\left(\frac{\lambda_0 - \lambda_1}{\lambda_1}\right)$ because the grooves discharge not only at their ends but also sideways for some distance back from the ends; and, in addition, the transition is not completed for some distance into the ungrooved zone. The length of the flow transition region is a function of the rib width relative to the groove width (γ) the relative flow resistance of the grooved and ungrooved passages, and the actual width of the rib.

The limitation of λ_1 to 0.1 seems rather arbitrary and results in unduly high pressures at the ends of the grooves (when $\epsilon = 0$). The difficulty is further aggravated in the case of compressible flow. A more realistic restriction would limit the ratio $\frac{\lambda_1 R}{h}$ to a value which assures the predominance of viscous over body forces in the flow of the fluid through the ungrooved passage.

Narrow and deep grooves are often advantageous although their mathematical treatment may be a bit difficult due to the introduction of significant sidewall and turbulence effects on the groove stream. Optimization for deep or irregular groove configurations can be readily accomplished when it is recognized that insofar as optimization is concerned, the primary criterion is the relative pressure at the ends of the grooves. This pressure should be on the order of 40 to 50 percent of the total pressure drop (when the shaft is centered). For greatest load capacity, the ratio $\frac{\lambda_0 - \lambda_1}{\lambda_1}$ is maximized; but for economy of flow, this ratio may be decreased and the relative pressure increased somewhat to offset losses due to increased lateral flow.

It should be noted with respect to Fig. 2, that the tapered journal bearing was introduced in the United States by Meuller[9] in 1958.

The reference to Messrs. Mannan, Fowler, and Carpenter is appreciated, however, in view of their excellent analysis of this interesting bearing.

Author's Closure

In answer to Mr. Adams, the functions of a step in the Adams bearing, Fig. 1, and of external restrictions in conventional, externally pressurized bearings are almost identical. However, a step is *not* an external restriction. External restrictions are situated outside lubricant film or clearance space. Such restrictions (radially directed holes, orifices or slits in the bearing bushing) are not present in the bearings of Figs. 1, 2, or 3. Indeed, such restrictions are superfluous. It is hoped that the meaning of the opening sentence of the paper is now understood.

The effect of elimination of short circuiting flow is shown in the following comparison:

(*a*) Maximum stiffness for a stepped journal bearing; see reference [3]

$$\gamma = 1; \quad \delta = 2.37; \quad \lambda_1 = 0.1; \quad \lambda_0 = 1; \quad \frac{p_m}{\epsilon \cdot \Delta p} = -0.268$$

and

$$\frac{p_m{}^2 h_0{}^3}{\epsilon^2 Q \mu} = 0.0115$$

(*b*) Maximum stiffness for a partly grooved journal bearing, see Table 1

$$\gamma = 0.1; \quad \delta = 4.94; \quad \lambda_1 = 0.1; \quad \lambda_0 = 1; \quad \frac{p_m}{\epsilon \cdot \Delta p} = -0.415$$

and

$$\frac{p_m{}^2 h_0{}^3}{\epsilon^2 Q \mu} = 0.0278$$

These data clearly show that stiffness and power-consumption improve. The data do not show yet to what extent short-circuiting flows are eliminated in the grooved bearing. Therefore, such flows were deliberately set to zero in the mathematical treatment, and the stiffness was again computed:

$$\gamma = 0.1; \quad \delta = 4.94; \quad \lambda_1 = 0.1; \quad \lambda_0 = 1; \quad \frac{p_m}{\epsilon \cdot \Delta p} = 0.422$$

The stiffness is now less than 2 percent higher than the stiffness computed when short-circuiting flows are accounted for. Thus it may be concluded that short-circuiting flows are eliminated.

Data collected in Table 1 are intended for the design of bearings when using incompressible lubricants. Such bearings can be expected to be stable. The construction parameters might be useful also for the design of gas bearings, see reference [3]. However, data collected in reference [3] and the present paper were not intended to give any information on stability. It might well be possible that better stability obtains with groove parameters differing from those in Table 1.

The author did *not* state that the grooved journal bearing is less costly than the stepped journal bearing. He did state that grooved journal bearings are less costly than conventional types with external restrictions. When comparing costs of stepped journal bearings and grooved journal bearings, it seems likely that the difference in cost will not be great. In this respect, it must be mentioned that local deviations from design values of groove width and groove depth (say 20 and 10 percent, respectively) are not expected to give anomalous results.

Actual test results of these grooved bearings using incompressible or compressible lubricants will be published in due time. Applications using incompressible lubricants depicted in Figs. 8 and 9 have operated very successfully.

The author appreciates that the discussion by Mr. Arneson reveals confidence in the author's approach and, at the same time, new information on partly grooved bearings.

In summary, more data are needed on:

1 Operation at greater eccentricities.

2 Operation with compressible lubricants, stability analysis.

3 Grooved bearings having less than about 15 grooves.

4 The effect of turbulence, fluid-inertia, and closer side-walls.

To these might be added:

5 Properties of these bearings when used as hybrid bearings.

6 The effect, on load capacity and stability, of grooves, inclined with respect to the direction of sliding; which forms an extension of work described in reference [5].

7 The effect of nonuniform groove parameters.

Particularly with gas lubrication, load capacity will show an increase if the grooves progressively deepen or widen in flow direction. It should be noted, that the partly grooved, externally pressurized journal bearing was introduced by the author in 1965 in a discussion to Mannam's paper, reference [2].

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[8] Arneson, H., "Hydrostatic Bearing Structure," U. S Patent No. 3,305,282

[9] Meuller, "Air Film Bearing for Machine Tools," U. S Patent No. 3,030,744