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A Systematic Study of Turbulent Film Flow

Turbulent film flow theories can only be verified on the basis of a large number of experimental results. Since it will be useful to handle these experimental results more or less systematically and to get some idea of the amount of work yet to be done, the first objective of this paper is to set up a classification system for turbulent film flow experiments. The second objective is to verify the bulk flow theory on the basis of the limited number of experimental results available in the literature and to show this theory to be compatible with these results.

Introduction

Turbulent film flow theories can only be fully verified on the basis of a large number of experimental results. In this paper it will be argued that it is wise to limit the number of theories to be verified to one, and it will be shown why one is sufficient. In this task, it will be useful to organize experimental results systematically, and thereby to get some idea of the amount of experimental work yet to be done.

As far as theory is concerned, it is more difficult to predict pressure build-up and flow in a turbulent lubricant film than in a laminar lubricant film, since the Navier-Stokes equations, which are applicable to both types of lubricant film, cannot be sufficiently simplified in the case of the turbulent lubricant film to obtain a complete set of equations with direct solutions. Moreover, the existing and commonly used simplification method for turbulent flow (consisting in averaging the fluctuations in velocity, pressure and density or combinations thereof, over an acceptably large period of time and an acceptably large area) results in a number of additional unknown factors. In particular, the averages of products of flow velocity fluctuations cannot yet be determined theoretically. One might try to measure these in lubricant films for a large number of cases and to use the experimental data for substituting into, and thus solving, the Navier-Stokes equations. However, this method needs further development.

A prediction of pressure build-up and flow in a turbulent lubricant film is possible also on the basis of a theory developed by Constantinescu [1],[1] based on the classical *mixing-length concept.*

Another theory, termed *law-of-wall theory* (adapted by Elrod and Ng [2] to the turbulent lubricant film) is based on the hypothesis that there is a universal shape of the flow-velocity profile that is in the vicinity of the surface boundary when averaged in time and suitaby normalized.

The hypotheses of these two theories permit one to confine himself to a limited number of measurements of flow velocity profiles, to derive from them certain hypothetical constants,

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[1] Numbers in brackets designate References at end of paper.

Nomenclature

a = index indicating stationary surface
b = bearing width; index indicating sliding surface
d = diameter
$f(\cdot\cdot)$ = functional relationship
h = film thickness
h_0 = radial clearance
$m_0; m_1$ = constants, see formulas (1), (2)
$n_0; n_1$ = constants, see formulas (1), (2)
p = pressure
p_m = mean pressure per unit projected bearing area
R_e = Reynolds number based on characteristic velocity
r_1 = pipe radius
r = radius of curvature, radius of cylindrical bearing
U = sliding velocity
u_m = mean velocity of flow relative to stationary surface
u_x = mean velocity in x direction
u_y = mean velocity in y direction
$U_x = \frac{u_x}{U}$ = dimensionless mean velocity of flow in x direction
$U_x = \frac{u_y}{U}$ = dimensionless mean velocity of flow in y direction
x, y = coordinates in sliding direction and at right angles to sliding direction in the plane of and attached to the stationary surface
τ = shear stress at a surface
τ_0 = ditto due to flow under the influence of a pressure gradient
τ_1 = ditto due to the sliding of a surface
ρ = density
η = viscosity
$\epsilon = \frac{e}{h_0}$ = dimensionless eccentricity
$R = \frac{\rho U h}{\eta}$ = Reynolds number based on sliding speed

and then to calculate flow velocity profiles for all combinations of the two components of flow. These components are "pressure flow" under the influence of a pressure gradient, and "drag flow" due to the sliding of a surface. Burton [3] concludes that both calculating methods eventually yield flow velocity profiles which do not depart too much from the actual ones. It may be added here that experiments by Orcutt [4] and others with a tilting-pad bearing show that the load-carrying capacity can be accurately predicted by the law-of-wall theory. Judging from Constantinescu's findings [1, 5], reasonable agreement with experimental results can probably also be achieved by using the mixing-length concept.

Burton [3] arrived at a simpler theoretical approach by interrelating all basic characteristics of a lubricant film, such as pressure gradient, sliding velocity of a surface, and shear stresses at either surface, to a characteristic velocity of flow in the lubricant film, *the mid-channel velocity*, defined as the one at the midplane between the two surfaces. He succeeded in doing this by using simplified time-averaged, flow-velocity profiles. Computations based on this method have been confined to simple types of self-acting bearings and an extension of this theory to externally pressurized and hybrid bearings would appear difficult.

All of the four aforementioned theoretical approaches are based on information obtained from experiments regarding:

(*a*) fluctuating velocity components due to turbulence

or

(*b*) velocity profiles so time-averaged as to eliminate fluctuating velocity components.

It has been sufficiently demonstrated in the foregoing that this approach is subject to difficulties in measuring velocities of flow and in processing the experimental data. Therefore, the author has made an attempt at a sufficiently accurate description of pressure build-up and flow in a lubricant film that is based on correlational data about bulk flow relative to each of the two bearing surfaces. It has been proved possible to develop a theory on the basis of such information and to calculate the properties of quite a variety of bearing types: self-acting, hybrid as well as externally pressurized bearings, Hirs [6, 7].

This bulk-flow theory is based on an analogy between turbulent flow under the influence of a pressure gradient and also due to the sliding of a surface. It had already been found by Davies and White [8] and Couette [9], respectively, that in either type of turbulent flow the wall shear stress depends on density, viscosity, mean flow velocity with respect to the particular surface for which the shear stress is considered, and thickness of the fluid film. Combining their findings, it can be shown that in either type the representation of this dependency requires, as a minimum, two dimensionless groups, e.g., a friction factor and a Reynolds number.

When plotting these dimensionless groups against each other, it is striking that in the turbulent regime the curve representing experiments for pressure flow is remarkably close to that representing drag flow.[2] Thus, it can be concluded that the dependency of the two dimensionless groups and, therefore, the dependency of wall shear stress on (*a*) density, (*b*) viscosity, (*c*) mean flow velocity with respect to the surface concerned, and (*d*) film thickness is fairly insensitive to the type of lubricant film flow. The author has found evidence that these dependencies are also insensitive to fairly general kinds of combinations of the two flow components such as mutually perpendicular flows, one being a pressure flow and the other a drag flow. It is the insensitivity of wall shear stress to the type of flow that has led the author to a treatment of flow in lubricant films, termed the bulk flow theory. A summary of this theory, including basic equations, will be given in the next section.

[2] Burton [4] has found an even closer agreement between the two flow types when using mid-channel velocity instead of mean flow velocity.

For verifying the bulk flow theory, it will be useful to handle experimental results more or less systematically and to get some idea of the amount of work yet to be done. For this purpose, the following classification system has been set up:

Types of Film Flow

0001 "Pressure flow" under the influence of a pressure gradient
0002 "Drag flow" due to the sliding of a surface
0003 A combination of the two main types of flow in parallel directions
0004 A combination of the main types of flow in directions including an angle.

Lubrication Film Profile

0010 Plane lubricant film, uniform film thickness
0020 Curved lubricant film, uniform film thickness
0030 Plane lubricant film, nonuniform film thickness
0040 Curved lubricant film, nonuniform film thickness

Surface Finish

0100 Both surfaces smooth
0200 One surface smooth, the other being rough
0300 Both surfaces rough.

Small Scale Design Characteristics of the Surfaces

1000 Both surfaces ungrooved
2000 One surface ungrooved, the other having a large number of shallow grooves
3000 Both surfaces have a large number of shallow grooves.

On the basis of this classification system it is possible to give a systematical survey of experimental data found in the literature. Almost all data will be seen to be characteristic of smooth ungrooved surfaces (1100). Within this category, sixteen combinations of types of flow and film profile can be formed on the basis of the above classification system. Nine combinations can be found in the literature. It will be seen that they are the more significant combinations out of the sixteen possible combinations. However, enough experimental data for strongly non-parallel surfaces are lacking. One of the combinations, i.e. 1134, is particularly important because practical bearings fall in this category.

The remaining two experiments, one with rough and the other with grooved surfaces, do not suffice for generally proving the bulk flow theory. However, with grooved surfaces the experiment for a complicated combination of the two main flow types (2124) is extremely important.

Bulk Flow Theory and Basic Equations

For a brief outline of the bulk flow theory it is useful to realize that there exists a similarity of the two main types of flow:

0001 "pressure" flow under the influence of a pressure gradient and
0002 "drag" flow due to the sliding of a surface.

For pressure flow it has been found that

$$\frac{\tau_0}{{}^1/_2\,\rho u_{m_0}{}^2} = n_0 \left(\frac{\rho u_{m_0} h}{\eta}\right)^{m_0} \tag{1}$$

For drag flow, it has been found that

$$\frac{\tau_1}{{}^1/_2\,\rho u_{m_1}{}^2} = n_1 \left(\frac{\rho u_{m_1} h}{\eta}\right)^{m_1} \tag{2}$$

where $u_{m_1} = {}^1/_2 U$, this being the mean flow velocity due to the sliding surface speed U. The similarity is not only evident from the fact that the two relationships for the surface shear stress have a similar form but also from the fact that the values for n_0 and n_1 and for m_0 and m_1 differ but little. Indeed, it can be shown that

$$n_0 = 1.2n_1 \text{ and } m_0 = m_1 \qquad (3), (4)$$

The foregoing two equations are at the basis of the development of the bulk flow theory. Eventually, more general equations for surface shear stress and pressure build-up result. Only those for the pressure build-up are presented here:

$$-\frac{h^2}{\eta U}\frac{\partial p}{\partial x}\left(\frac{\eta}{\rho U h}\right)^{1+m_0} = {}^1/_2 n_0 \left[U_x(U_x^2 + U_y^2)^{\frac{1+m_0}{2}} + (U_x - 1)^2\{(U_x - 1)^2 + U_y^2\}^{\frac{1+m_0}{2}} \right] \qquad (5)$$

$$-\frac{h^2}{\eta U}\frac{\partial p}{\partial y}\left(\frac{\eta}{\rho U h}\right)^{1+m_0} = {}^1/_2 n_0 \left[U_y(U_x^2 + U_y^2)^{\frac{1+m_0}{2}} + U_y\{(U_x - 1)^2 + U_y^2\}^{\frac{1+m_0}{2}} \right] \qquad (6)$$

where $U_x = \frac{u_x}{U}$ and $U_y = \frac{u_y}{U}$ are normalized velocities of flow, and coordinate system xy is attached to the stationary surface, x in the direction of sliding, y perpendicular to the direction of sliding, u_x mean flow velocity with respect to coordinate system in the direction of sliding, u_y mean flow velocity with respect to coordinate system perpendicular to the direction of sliding.

In general the above equations are nonlinear as far as bulk flow velocities are concerned. However, nonlinearity is restricted to the turbulent flow regime, where $-0.25 < m_0 < 0$. For the special case of laminar flow the equations become linear:

$$m_0 = -1 \text{ and } n_0 = 12$$

$$-\frac{h^2}{\eta U}\frac{\partial p}{\partial x} = 12\,(U_x - {}^1/_2) \qquad (7)$$

$$-\frac{h^2}{\eta U}\frac{\partial p}{\partial y} = 12 U_y \qquad (8)$$

There are also special cases in the turbulent flow regime where equations (5) and (6) become linear. One of these cases is characteristic of self-acting bearings operating with slightly nonparallel bearing surfaces. Then, the pressure flow component is much smaller than the drag flow component and the following two equations result:

$$-\frac{h^2}{\eta U}\frac{\partial p}{\partial x} = \frac{n_0(2 + m_0)}{2^{1+m_0}}\left(\frac{\rho U h}{\eta}\right)^{1+m_0}(U_x - {}^1/_2) \qquad (9)$$

$$-\frac{h^2}{\eta U}\frac{\partial p}{\partial y} = \frac{n_0}{2^{1+m_0}}\left(\frac{\rho U h}{\eta}\right)^{1+m_0} U_y \qquad (10)$$

Equations (9), (10), and (7), (8) show a similar form. This similarity will be exploited in the following section, sub item 1134.

Systematic Survey of Experiments Verifying Bulk-Flow Theory

Following the system introduced earlier in this paper, experimental results are now discussed. Each item is characterized by a number; units indicate the type of film flow, tens indicate the film profile, hundreds indicate the surface finish and thousands the small scale design characteristics of the surfaces.

1111 Pressure Flow, Plane Lubricant Film and Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. Of the many experimental results obtained with turbulent flow between two surfaces, those of Davies and White [8] have been selected. From these tests the values of the constants in formula 1 relating friction-factor and Reynolds number can be derived. For Reynolds numbers

$$\frac{\rho u_m h}{\eta} \text{ smaller than } 10^5 \text{ we find:}$$

$$n_0 = 0.066 \text{ and } m_0 = -0.25.$$

In a book by Schlichting [10] and a survey article by Hartnett, Koh and McComas [11], it was shown that the hydraulic diameter concept is valid when comparing experimental results with pipes and between two surfaces. This concept enabled the author to derive the same two values given above for the experimental constants n_0 and m_0 from experiments on turbulent flow in the annular space formed by two concentric round pipes (Koch and Feind [12]) and from the basic experiments on flow in round pipes by Blasius [13]. All the experiments cited here indicate that the above two values for n_0 and m_0 are valid up to Reynolds numbers of 10^5. At higher Reynolds numbers slightly different values will be found.

1112 Drag Flow, Plane Lubricant Film, and Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. Two sets of tests, one by Couette [9] and the other, by Robertson [14], come in this category. The results of these two studies agree surprisingly well with each other. The values of the constants in formula 2 can be derived from these tests. Up to the highest Reynolds number at which these measurements were made, i.e. $\frac{\rho u_m h}{\eta} \approx 30{,}000$, we find: $n_1 = 0.055$ and $m_1 = -0.25$. True, on the sole basis of the experiments by Robertson, the value for m_1 might be smaller: $m_1 = -0.2$. However, the author felt that the range of Reynolds numbers covered by Robertson 10^4–3×10^4 was not sufficiently wide and that values for n_1 and m_1 should rather be so taken as to yield the best correlation for the entire range covered together by Robertson [14] and Couette [9].

It is to be observed that only Robertson used a plane film. Couette used a stationary shaft and a rotating bearing with a small radial clearance to radius ratio. But in such cases turbulent Couette flow is physically almost identical to turbulent plane flow.

Robertson's measurements of the friction factor are indirect, being derived from measured flow velocity profiles. Couette's measurements are direct in that he measured the torque exerted on the shaft.

1113 Combination of the Two Main Types of Flow in Parallel Directions; Plane Lubricant Film and Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. Only the results derived from Shinkle and Hornung's [15] test series are given here. In their test series the flow relative to the stationary surface was blocked by a barrier attached to the stationary surface. The shear stress $\tau_b = \tau_0 - \tau_1$ at the sliding surface (τ_0 due to the pressure flow component τ_1 due to the drag flow component) was measured. Their results are given in a graph and they can very well be represented by the following formula:

$$\frac{\tau_b}{{}^1/_2\,\rho U^2} = -0.062\left(\frac{\rho U h}{\eta}\right)^{-0.25} \qquad (11)$$

for $3.000 < \frac{\rho U h}{\eta} < 30{,}000$. A relation between shear stress τ_0 and τ_1 can be found by looking at formulas 1 and 2 and accounting for the bulk flow velocity with respect to the stationary surface being absent. The absence of bulk flow velocity indicates that

$$u_{m_0} + u_{m_1} = 0$$

and, consequently,

$\tau_0 + n_0/n_1\,\tau_1 = 0$ where according to 1111 and 1112 $n_0 = 0.066$ and $n_1 = 0.555$

Thus the shear stress at the sliding surface $\tau_b = \tau_0 - \tau_1$ can be rewritten as follows

$$\tau_b = 1.8\tau_0$$

where $\tau_0 = -\frac{1}{2}\frac{dp}{dx}h$ and is the shear stress due to a pressure gradient. The above experimental formula 11 now becomes

$$\frac{h^2}{\eta U}\frac{dp}{dx}\left(\frac{\eta}{\rho U h}\right)^{0.75} = 0.034$$

The same expression can be derived from formula 5 by inserting $U_x = U_y = 0$:

$$\frac{h^2}{\eta U}\frac{dp}{dx}\left(\frac{\eta}{\rho U h}\right)^{1+m_0} = {}^1/_2 n_0$$

where m_0 was found to be -0.25 and $n_0 = 0.066$ in item 1111. Thus the agreement of 1113 (pressure and drag flow in the same direction) with more specialized and more general cases is excellent.

It should be noted that for $\frac{\rho U h}{\eta} > 30{,}000$, Shinkle and Hornung's test series showed the influence of measuring errors, of roughness of the surface, or of inertia effects in the flow other than those inherent in turbulence. Consequently, at $\frac{\rho U h}{\eta} =$ 100,000 the theoretical value of 15 percent lower than the experimental value. However, in the aforementioned regime of Reynolds numbers greater than 3000 and smaller than 30,000 excellent agreement has been found.

Strictly speaking, the experiments here surveyed do not come into category 1113 because the lubricant film had a curved profile. However, in Shinkle and Hornung's test series the effect of the curvature was negligible because the mean velocity of flow in the circumferential direction was zero: $u_x = 0$.

1121 Pressure Flow, Curved Lubricant Film and Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. No experiments are known that are directly related to the present case. However, the many measurements published for curved pipes enable us to gain an insight into the effect of curvature of the surfaces on the flow, particularly into the inertia effects associated with the curvature. It will be evident that we need only consider those cases in which the radius r_1 of the pipe is small with respect to the radius r of curvature. In bearings, the ratio between the lubricant film thickness and the radius of curvature will invariably be exceedingly small ($\sim 0.003 < h/r < \sim 0.03$).

According to tests by White [16] and Ito [17], the ratio between pressure drop with flow in a curved pipe and in a straight one can, for the range covered by them, be expressed by:

$$a_0 + a_1\left(\frac{\rho u_m r_1}{\eta}\right)^{1/4}\left(\frac{r_1}{r}\right)^{1/2}$$

The values found for a_0 and a_1 do not completely agree in these two test series. For our purpose it is sufficient to know that $a_0 \sim 1$ and $a_1 \sim 0.1$. It can then be derived immediately that for

$$\frac{\rho u_m r_1}{\eta} < 10^5 \text{ and } \frac{r_1}{r} < 0.03$$

the curvature of the pipe causes an increase of pressure drop not exceeding 30 percent. Keeping in mind that the Reynolds numbers and the film thickness to radius ratio in bearings certainly do not reach the above values, we will in all the cases still to be described, in which the curvature of surfaces also plays a role, try to demonstrate that the effect of the curvature is small.

1122 Drag Flow, Curved Lubricant Film and Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. Three test series come into this category. The authors are Burton [3], Taylor [18], and Pan and Vohr [19]. Tests by Burton [3] with a bearing model with the rather large ratio of 0.016 between lubricant film thickness and radius include, among other things, indirect measurements of the shear stress at a surface due to drag flow. From his tests it can be concluded that for

$$1000 < \frac{\rho u_m h}{\eta} < 6000$$

the constants of formula 3 had the following values:

$$n_1 = 0.065 \text{ and } m_1 = -0.25.$$

As regards Burton's tests with the lower values of these Reynolds numbers, the constants, substituted in formula 2 gave values somewhat too low for the shear stress at the surface, i.e., down to -10 percent. Probably these low values are due to an influence of Taylor vortices. Conditions for Taylor vortices to occur in a transition region between laminar and turbulent flow were favourable because Burton, as well as the other authors, used a rotating journal and a stationary bearing.

Taylor [18] used a still greater lubricant film thickness to radius ratio in his tests, viz., $\frac{h}{r} = 0.028$. From his results the same values can be derived, viz., $n_1 = 0.065$ and $m_1 = -0.25$ for

$$\frac{\rho u_m h}{\eta} > 5000.$$

Finally, tests by Pan and Vohr [19] should be mentioned, these showing the highest film thickness to radius ratio (much higher than the value which is desirable for bearings), viz., $\frac{h}{r} = 0.099$. From their tests we can derive: $n_1 = 0.085$ and $m_1 = 0.25$ for

$$\frac{\rho u_m h}{\eta} > 5000.$$

It should also be noted that both from Taylor's and from Pan and Vohr's tests it follows that up to approximately

$$\frac{\rho u_m h}{\eta} = 5000,$$

Taylor vortices may have had a considerable influence and that the shear stresses which can be predicted on the basis of the aforementioned constants, i.e., for $\frac{\rho u_m h}{\eta} < 5000$, will be lower than the measured values. On the other hand, in Burton's tests the influence of vorticity was much less pronounced in this same range. Consequently, there might well have been some mechanism that suppressed vorticity in Burton's tests and due to which the transition to turbulent flow took place at lower Reynolds numbers, somewhere in the neighborhood of $\frac{\rho u_m h}{\eta} \sim 1000$. A different explanation might lie in the fact that Burton derived shear stresses indirectly, namely from measured flow velocity profiles and that the influence of vorticity on shear stress becomes rather elusive by doing so.

The general conclusion is that the effect of the curvature is small and manifests itself in an increase of the constant n_1 by less than 20 percent if $\frac{h}{r} < 0.03$ and that the increase may even be neglected altogether whenever $\frac{h}{r} \leq 0.003$. The values for the constant m_1 remain unchanged. If Taylor vortices are present in the films, an estimate of the shear stress on the basis of the above values for constants n_1 and m_1 tends to be conservative. However, the influence of these vortices on shear stress will be confined to Reynolds numbers smaller than 5000.

1123 Combination of the Two Main Types of Flow in Parallel Directions Curved Lubricant Film, Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. In the literature the author could find only one test (Burton [3]) in the present category. Burton has measured pressures in a flow relative to the stationary surface; the flow was not entirely blocked, and u_m amounted to about $^1/_4$ U. His test was made under conditions in which inertia effects at the inlet and outlet of the film played a considerable role.

Notwithstanding these inertia effects, it can be derived from a measurement at one Reynolds number that:

$$\frac{h^2}{\eta U}\frac{dp}{dx}\left(\frac{\eta}{\rho U h}\right)^{1/4} = 0.021 \pm 10 \text{ percent}$$

By substituting $u_x = {}^1/_4 U$, $u_y = 0$, and $n_0 = 0.066$, $m_0 = -0.25$[3] in formula 5 and by rearranging, it follows that

$$\frac{h^2}{\eta U}\frac{dp}{dx}\left(\frac{\eta}{\rho U h}\right)^{1/4} = 0.017.$$

Hence the theoretical result obtained when using values for n_0 and m_0 valid for plane films is somewhat lower than the experimental one. This might well be due to the fact that in Burton's experiment [3] a shaft was rotating in a bearing with the rather great radial clearance to radius ratio of 0.016. Hence, the influence of the inertia effects induced by the curvature of the lubricant film again manifests itself in a moderate increase in pressure build-up, i.e., by about 20 percent.

1124 Combination of the Two Main Types of Flow in Directions Including an Angle, Curved Lubricant Film and Uniform Film Thickness, Both Surfaces Smooth and Ungrooved. Particular attention is given here to the combination of pressure flow and drag flow in which both flow-components are at right angles to each other. This flow pattern occurs, for instance, when a shaft is made to rotate concentrically in a cylindrical bearing and when a flow in the axial direction between the surfaces is set up by an axial pressure gradient. This flow pattern was realized in tests by Tao and Donovan [20] and by Yamada [21]. Only Yamada's tests are treated here because they seem to be more reliable and are interpreted more easily. Yamada represents the results of his measurements as follows:

$$\frac{\frac{dp}{dy}h}{\rho u_y^2} = f\left(\frac{\rho U h}{\eta}, \frac{\rho u_y h}{\eta}\right)$$

with u_y in the axial direction y, and U in the circumferential direction x. At neither surface did Yamada measure shear stresses in the circumferential direction. His tests were focused on measuring axial flow rate and axial pressure gradient at various rotational speeds.

From formula 6, it is possible to derive an expression which lends itself to a comparison with Yamada's experimental results.

To this end the formula should first be adapted to the present condition where the average flow component in the circumferential direction must have been equal to half the sliding velocity: $U_x = \frac{u_x}{U} = 0.5$. This accounts for the fact that there is no pressure flow component in the circumferential direction. After some calculation it follows that:

$$\frac{\frac{dp}{dy}h}{\rho u_y^2} = -n_0\left(\frac{\rho u_y h}{\eta}\right)^{m_0}\left\{\frac{1}{4}\left(\frac{U}{u_y}\right)^2 + 1\right\}^{\frac{1+m_0}{2}}$$

or, by substituting $\lambda = \frac{4\frac{dp}{dy}h}{\rho u_y^2}$, $\text{Re} = \frac{\rho u_y h}{\eta}$

and $R_\omega = \frac{\rho U h}{\eta}$,

$$\lambda = 4\, n_0 \text{Re}^{m_0}\left\{\frac{1}{4}\left(\frac{R_\omega}{\text{Re}}\right)^2 + 1\right\}^{\frac{1+m_0}{2}}$$

For sub category 1111, the following values for n_0 and m_0 with pressure flow have been found; $n_0 = 0.066$ and $m_0 = -0.25$. However, Yamada [21] has found slightly different values: $n_0 = 0.065$ and $m_0 = -0.24$ for his particular experimental apparatus with pressure flow ($R_\omega = 0$). This discrepancy is probably due to a slight influence of inertia effects other than those inherent in turbulence. In Fig. 1, some of Yamada's experimental results have been depicted. It can be seen clearly that these results agree roughly with the above equation. For a few cases depicted in Fig. 1 a closer inspection of the agreement between theory (dotted lines) and experiment (drawn lines) is possible. The dotted, theoretical lines apply to the cases characterized by $R_\omega = 3000$ and 5000 in Fig. 1(*a*) and $R_\omega = 10{,}000$ and 20,000 in Fig. 1(*b*) and are based on the above equation when substituting therein Yamada's values for n_0 and m_0. It can be seen that the agreement between theory and experiment is excellent for $R_\omega \gg \text{Re}$ and $\text{Re} \gg R_\omega$, or $U \gg u_y$ and $u_y \gg U$. But in a regime characterized by $\text{Re} \approx {}^1/_2 R_\omega$, or $u_y \approx {}^1/_2 U$, his experimental results are generally higher (about 20 percent) than theoretical results. This effect can probably be attributed to Taylor vortices.

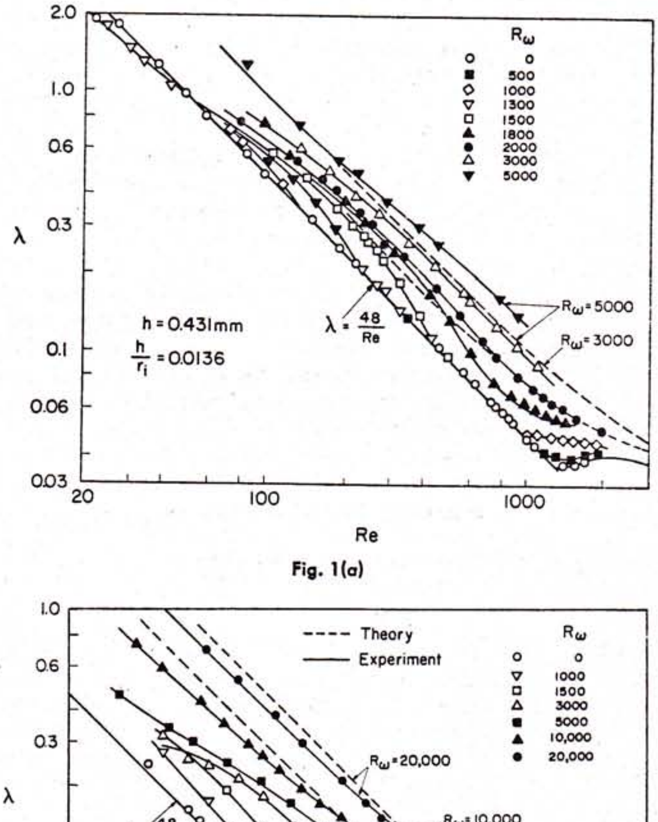

Fig. 1(a)

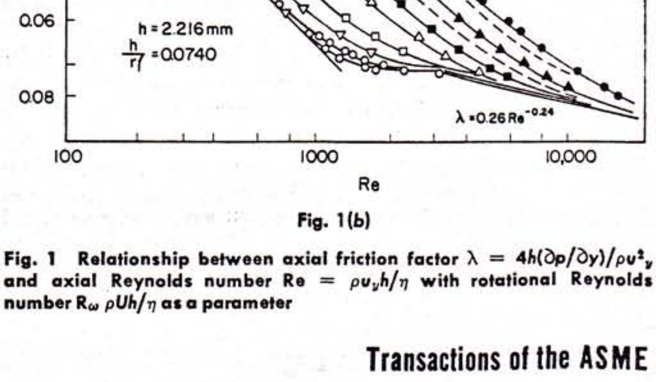

Fig. 1(b)

Fig. 1 Relationship between axial friction factor $\lambda = 4h(\partial p/\partial y)/\rho u_y^2$ and axial Reynolds number $\text{Re} = \rho u_y h/\eta$ with rotational Reynolds number R_ω $\rho U h/\eta$ as a parameter

[3] See sub 1111.

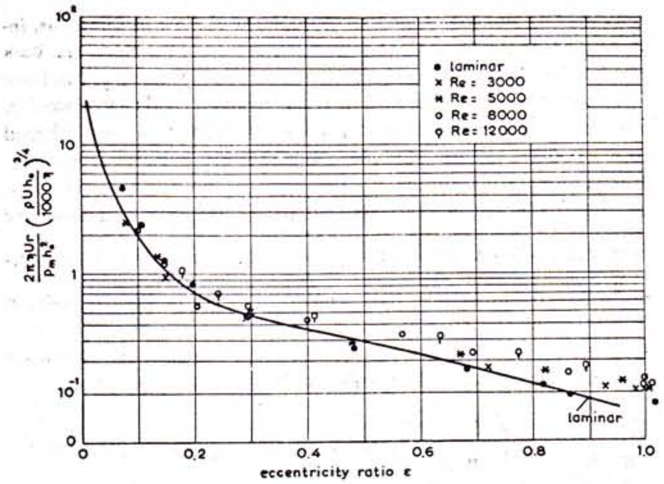

Fig. 2(a)

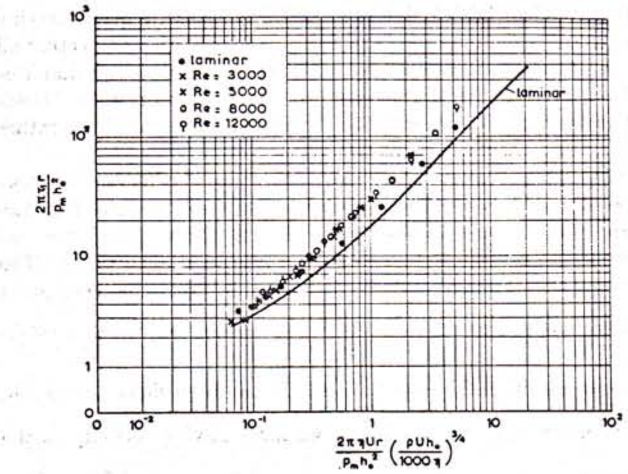

Fig. 2(b)

Fig. 2 Replot of Orcutt's experimental results for tilting pad bearings. Graph *a* shows the relationship between the dimensionless load carrying capacity (a combination of Sommerfeld number and Reynolds number) and eccentricity. Graph *b* shows the relationship between dimensionless frictional torque and the Sommerfeld-Reynolds number. The Sommerfeld-Reynolds number which is characteristic of turbulent flow is reduced to the Sommerfeld number characteristic of laminar flow by inserting $\rho U h_0/\eta = 1000$.

1134 Combination of the Two Main Types of Flow in Directions Including an Angle, Plane Lubricant Film and Nonuniform Film Thickness, Both Surfaces Smooth and Ungrooved. A number of experiments are known with plain cylindrical fluid film bearings (Smith and Fuller [22] and Ketola and Mc Hugh [23]), and with tilting-pad bearings (Orcutt [4]). All these experiments may be used to verify the theory.

In these tests the film thickness to radius ratio was so small that the curvature of the lubricant film and inertia effects due to the wedge shape of the film may be neglected: $\frac{h}{r} \approx 10^{-3}$. It can be derived from equations (5) and (6), plus the continuity condition that a solution of these equations should give a result of the following nature:

$$\frac{p_m h_0^2}{\eta U r} = \left(\frac{\rho U h_0}{\eta}\right)^{1+m_0} f_1\left(\epsilon, \frac{b}{d}\right)$$

where

p_m = mean pressure over the projected surface bd
h_0 = radial clearance
ϵ = dimensionless eccentricity $\frac{e}{h_0}$
$\frac{b}{d}$ = width/diameter ratio.

For the limiting case where that the eccentricity is small and, thus, the drag flow is dominant the above formula can also be derived from equations (9) and (10) plus the continuity condition.

For laminar flow in the lubricant film it is well known that:

$$\frac{p_m h_0^2}{\eta U r} = f_2\left(\epsilon, \frac{b}{d}\right).$$

The above formula can be derived from equations (7) and (8) plus the continuity condition.

As expected, there appears to exist a critical Reynolds number:

$$\left(\frac{\rho U h_0}{\eta}\right)_c$$

at which the two equations yield identical numerical results. This critical number represents the transition from laminar to turbulent flow as far as the calculation of load-carrying capacity is concerned. The value for this critical Reynolds number turns out to be:

$$\left(\frac{\rho U h_0}{\eta}\right)_c \approx 1000 - 2000$$

The lower value is representative of wide bearings where the pressure flow is mainly in the drag flow direction. This lower value is found by equating equations (7) and (9) and inserting $n_0 = 0.066$ and $m_0 = -0.25$. The higher value is representative of narrow bearings where the pressure flow is mainly perpendicular to the drag flow direction. This upper value is found by equating equations (8) and (10).

By making use of the present concept of a critical or transitional Reynolds number, an attractive simplification of numerical work can be obtained. Indeed, the formula for load-carrying capacity in the case of turbulent flow in the lubricant film can profitably be rewritten as follows:

$$\frac{p_m h_0^2}{\eta U r} = \left(\frac{\rho U h_0}{\eta}\right)^{1+m_0} \left(\frac{\rho U h_0}{\eta}\right)_c^{-(1+m_0)} f_2\left(\epsilon, \frac{b}{d}\right)$$

in which the functional relationship $f_2\left(\epsilon, \frac{b}{d}\right)$ is known from laminar lubrication theory, in which $m_0 = -0.25$ for smooth surfaces and in which $\left(\frac{\rho U h_0}{\eta}\right)_c \approx 1000$ for small eccentricities, wider bearings and smooth surfaces.

It proves indeed that the above formula roughly agrees with the results of two of the three test series. Excellent agreement can be obtained with Orcutt's experiments. Making use of the above formula and of a critical value for the Reynolds number $\left(\frac{\rho U h_0}{\eta}\right)_c \approx 1000$, leads to a replot of his graphs. The relationship between the dimensionless load-carrying capacity and dimensionless eccentricity is shown in Fig. 2(a) and the relationship between dimensionless frictional torque and dimensionless load-carrying capacity in Fig. 2(*b*). It can be seen in Figs. 2(*a*) and 2(*b*) that all experimental results can be represented by one line and that this line is close to that for the laminar experimental results. Deviations can be seen to occur only if eccentricity tends to be rather high.

1143 Combination of the Two Main Types of Flow in Parallel Directions, Curved Lubricant Film and Nonuniform Film Thickness, Both Surfaces Smooth and Ungrooved. Pan and Vohr [19] give experimental results pertaining to the foregoing conditions. The bearing tested was a cylindrical fluid film bearing of very great width; the maximum value of $\frac{\rho U h_0}{\eta}$ was 3200 and the film thickness to radius ratio $\frac{h_0}{r} = 0.0104$. It has been demonstrated in the foregoing that in this case we have Taylor vorticity in the lubricant film. Above a given Reynolds number $\frac{\rho U h_0}{\eta}$ the flow becomes fully turbulent and the effect of the curvature only results in an increase of n_0 and n_1 in formulas 1 and 2 (see also sub 1121 and 1122). The test results make it possible to determine the Reynolds number $\frac{\rho U h_0}{\eta}$ for which the effect of vorticity on pressure build-up is negligible. For this purpose, we use those test results in which the ratio of the peak pressures in the lubricant film with turbulent and laminar flow is given for Reynolds numbers $\frac{\rho U h_0}{\eta} < 3200$. The author has found the following values for the infinitely wide bearing:

For $\epsilon = 0.2$

$$(a)\quad \text{turbulent:}\quad \frac{p_{\max} h_0^2}{\eta U r} = 0.007 \left(\frac{\rho U h_0}{\eta}\right)^{3/4}$$

$$(b)\quad \text{laminar:}\quad \frac{p_{\max} h_0^2}{\eta U r} = 1.23$$

so that:

$$\frac{p_{\max\ \text{turbulent}}}{p_{\max\ \text{laminar}}} = 0.0057\left(\frac{\rho U h_0}{\eta}\right)^{3/4}$$

Comparison of this expression with the experimental results shows that for $\frac{\rho U h_0}{\eta} > 2400$ there is good agreement and that the estimate on the basis of the theory is less than 10 percent lower than the test result.

1311 Pressure Flow, Plane Lubricant Film, and Uniform Film Thickness, Both Surfaces Rough and Ungrooved. So far it has been attempted to make the bearing surfaces as smooth as possible. If the flow in the lubricant film is laminar this is no doubt sensible and the roughness of the surfaces has a negligible effect on pressure build-up in and leakage from the lubricant film. Hence roughness indicates only that the surfaces will be in contact sooner than for the case of smooth surfaces.

In the case of turbulent flow in the lubricant film it is by no means self-evident that the surfaces should be given a smooth finish. Indeed, it is conceivable that roughness of even a small percentage of the lubricant film thickness may already result in a worthwhile increase in pressure build-up and/or in a reduction of leakage.

From the data compiled by Schlichting [10] it can be concluded that the constant m_0 in the formulas 1 to 10 increases with increasing heights of the roughness from a small negative value for smooth surfaces to $m_0 = 0$ for very rough surfaces.

It seems possible to achieve an increase in load-carrying capacity of self acting bearings by a factor of about 2 and a reduction of leakage in externally pressurized bearings by a similar factor 2. It seems that until now, no work pertinent to this improvement of bearing performance has been published.

2124 Combination of the Two Main Types of Flow in Directions, Including an Angle, Curved Lubricant Film, Uniform Film Thickness, Both Surfaces Smooth and Grooved. These conditions are realized in tests by Stair [24] and Pape [25] with grooved seals (viscoseals). The seal consists of a shaft with a multiple square screw thread in a smooth bushing; shaft and bushing are accurately concentric, and rotation builds up pressure in the axial direction *y*. In Stair's tests, the shaft rotated in the bushing and vorticity occurred up to certain values of $\frac{\rho U h}{\eta}$. In Pape's tests the bushing rotated around the shaft and there was a sudden transition from laminar flow to turbulent flow.

From equations (5) and (6) it can be shown that the following formula would have to be applicable in the turbulent regime:

$$\frac{h^2}{\eta U}\frac{dp}{dy} = n_0\left(\frac{\rho U h}{\eta}\right)^{1+m_0} f(\text{dimensionless groove parameters}),$$

in which $\frac{dp}{dy}$ gives the overall, smoothed axial pressure gradient, h gives the radial clearance, and where f denotes a functional relationship.

Despite the fact that inertia effects must have played a role, Stair's and Pape's tests again give the value $m_0 = -0.25$. Hence it appears that this constant never changes as long as $\frac{\rho U h}{\eta} < 10^5$ and as long as the surfaces are smooth, even though one of them is grooved.

The value of the constant n_0 could not be derived from the experiments because the function f of the dimensionless groove parameters has not yet been derived. However, from the experiments it seems possible to derive a critical Reynolds number where extrapolated laminar and turbulent results would yield equal pressure build-up.

Summary of Experimental Results

In Table 1, experimental results presented in the previous section have been summarized. The first column gives the system number. The second column gives a description of the turbulent film flow types. The third column gives values for the constants n and m used in equations (1)–(10). The fourth column gives ratios of film thickness h and radius of curvature r. The fifth and sixth columns give minimum and maximum values for the Reynolds numbers based on film thickness and bulk flow speed. The last column gives an estimation of the agreement between theory and experiments. The agreement is less good for the higher values of h/r in that differing values for n and m are found. However, for the lower values of h/r that are of importance to bearing design, the values for n_0 and m_0 and for n_1 and m_1 are consistent. Thus, sufficient proof for the bulk flow theory seems presented for bearings with the following characteristics:

000x any combination of pressure flow and drag flow

00x0 parallel and nonparallel surfaces, low curvature values (h/r)

0x00 both surfaces smooth

x000 both surfaces ungrooved and, probably, one surface ungrooved, the other having a large number of shallow grooves

More information is needed on bearings with rough and grooved surfaces.

Conclusion

1 The bulk flow theory has been shown to be reliable for predicting pressure build-up and flow in current turbulent bearings.

2 The reliability can be improved by increasing the number of basic experiments with current turbulent bearings.

3 Bearings with grooved and/or rough surfaces have a development potential and more experiments are needed to prove the bulk flow theory for such types.

Table 1

Classification		Experimental results		Ratio	Re_{min}	Re_{max}	Agreement theory experiment
1111	Pressure flow, plane lubricant film and uniform film thickness, both surfaces smooth and ungrooved	$n_0 = 0.066$	$m_0 = -0.25$	$\frac{h}{r} = 0$	10^3	10^5	good
1112	Drag flow, plane lubricant film and uniform film thickness, both surfaces smooth and ungrooved	$n_1 = 0.055$	$m_1 = -0.25$	$\frac{h}{r} = 0$	10^3	3×10^4	good
1113	Combination of the two main types of flow in parallel directions, plane lubricant film and uniform film thickness, both surfaces smooth and ungrooved	$n_0 = 0.066$ $n_1 = 0.055$	$m_0 = -0.25$ $m_1 = -0.25$	?	3×10^3	3×10^4	good
1121	Pressure flow, curved lubricant film and uniform film thickness, both surfaces smooth and ungrooved	$0.066 < n_0 < 0.088$	$m_0 = -0.25$	$\frac{h}{r} < 0.03$	10^3	10^5	fair
1122	Drag flow, curved lubricant film and uniform film thickness, both surfaces smooth and ungrooved	$n_1 = 0.065$ $n_1 = 0.065$ $n_1 = 0.085$	$m_1 = -0.25$ $m_1 = -0.25$ $m_1 = -0.25$	$\frac{h}{r} = 0.016$ $\frac{h}{r} = 0.028$ $\frac{h}{r} = 0.099$	10^3 5×10^3 5×10^3	6×10^3 2×10^4	fair
1123	Combination of the two main types of flow in parallel directions, curved lubricant film, uniform film thickness, both surfaces smooth and ungrooved	$n_0 = 0.066$	$m_1 = -0.25$	$\frac{h}{r} = -0.016$	2.4×10^3	2.4×10^3	good
1124	Combination of the two main types of flow in directions including an angle, curved lubricant film and uniform film thickness, both surfaces smooth and ungrooved	$n_0 = 0.052$	$m_0 = -0.24$	$\frac{h}{r} = 0.014$–0.115	10^3	2×10^4	fair
1134 bearing test	Combination of the two main types of flow in directions including an angle, plane lubricant film and non-uniform film thickness, both surfaces smooth and ungrooved	$\left(\frac{\rho U h}{\eta}\right)_c = 1{,}000$[a]	$m_0 = -0.25$	$\frac{h}{r} = 0.003$	2×10^3	1.2×10^4	good
1143	Combination of the two main types of flow in parallel directions, curved lubricant film and non-uniform film thickness, both surfaces smooth and ungrooved	$n_0 = 0.066$	$m_0 = -0.25$	$\frac{h}{r} = 0.01$	2.4×10^3	3.2×10^3	good
1311	Pressure flow, plane lubricant film and uniform film thickness both surfaces rough and ungrooved	$-0.25 < m_0 < 0$					
2124	Combination of the two main types of flow in directions including an angle, curved lubricant film, uniform film thickness, both surfaces smooth and grooved	$\left(\frac{\rho U h}{\eta}\right)_c \approx 500$[a]	$m_0 = -0.25$	$\frac{h}{r} = 0.004$	10^3	6×10^3	good

[a] This critical Reynolds number is a function of n_0, see sub 1134.

It is suggested that a complete experimental program be established following the classification system presented in the introduction. The bulk flow theory shows that measurements of wall shear stresses, pressure gradients, bulk flows and sliding speeds are sufficient. No information need be collected on flow velocity profiles for proving the bulk flow theory.

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